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# Memo

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**To:** Joseph Piacentini,
Employee Benefits Security Administration,
U.S. Department of Labor

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**Subject:** Generally Accepted Investment Theories

## 1. Introduction

It is our understanding that the Pension Protection Act of 2006 amends the prohibited transaction provisions of the Employee Retirement Income Security Act of 1974 (ERISA), and parallel rules under the Internal Revenue Code of 1986, to conditionally exempt transactions engaged in by a plan pursuant to advice provided under a computer based "eligible investment advice arrangement." Among other conditions, the advice must be provided pursuant to a computer model that "applies generally accepted investment theories that take into account the historic returns of different asset classes over defined periods of time.[1]

This document summarizes influential investment theories in modern finance literature. We start with a description of the foundations laid by Markowitz (1952) and a widely-cited application, the Capital Asset Pricing Model (CAPM). We next turn to several extensions of Markowitz (1952) and CAPM which by now have become widely used among finance scholars and practitioners. In particular, recognizing that the objectives of investors and their exposure to various risks may change over time, we discuss extensions which form the basis of dynamic portfolio allocation and reallocation over the life cycle.

For the purpose of this document, investment theories center on the optimal allocation of assets over investment options. The terms investment theory, portfolio allocation theory, and portfolio theory are used interchangeably.

[1] http://www.dol.gov/ebsa/pdf/ppa2006.pdf, pages 174-179.

## 2. The Literature

Investment options are characterized by at least two key features: expected rate of return and the degree of riskiness associated with the rate of return. There tends to be a trade-off between expected rate of return and riskiness. On one end of the spectrum are risk-free assets that promise a (relatively low) return without any risk. A close approximation to risk-free assets are U.S. government bonds. On the other end of the spectrum are assets with high expected rates of return and high levels of uncertainty. In other words, for such assets there is a good chance that the actual rate of return is substantially greater than the expected rate, but there is also a good chance that in any given period the actual return could turn out to be low or even negative. Examples of risky assets are shares in a technology start-up company and investments in art.

There are empirical regularities in rates of return across assets. The fortunes and misfortunes of some companies, perhaps driven by factors in a given industry or the broader economic market, tend to move together, so that their rates of return are positively correlated. Other assets' returns are negatively correlated, so that, if one asset experiences an above-average rate of return, the other tends to yield a below-average return. A key insight of modern finance theory is that these empirical regularities can be exploited to maximize a portfolio's expected rate of return without increasing risk, or minimize risk without sacrificing expected rate of return. Because different investments do well at different times, a portfolio constructed of multiple assets will generate portfolios with lower risk for a given level of return than individual assets. In an article that is widely considered the foundation of portfolio allocation theory, Markowitz (1952) derived the mathematics of optimal portfolio diversification.

### *Key Assumptions*

In optimal portfolio allocation theory, investors seek to maximize the return for a given level of risk, or find the lowest possible level of risk for a given average return. Markowitz (1952) relied on a simplifying assumption that is maintained in much of modern finance theory, namely that investors only care about the mean (expected) rate of return and its variance or standard deviation (risk). This assumption is known as the mean-variance paradigm. It considers risk only as the standard deviation of returns, and ignores skewness and other risk measures. For example, an investment of $100 which returns $91 or $111 with equal chances has an expected return of $1 and a standard deviation of $10. An investment of $100 which remains unchanged 99 percent of the time and returns $200 with a chance of 1 percent has the same mean and almost the same standard deviation. The mean-variance paradigm assumes that investors are indifferent between the two investment options. Several competing models account for more refined preference structures in which certainty of payout or downside potential play a role.

Another important assumption in investment theories is that investors know the distribution of potential rates of return. This assumption is typically implemented by equating the distribution of future returns to the distribution of historical returns.[2] Among others, portfolio allocation optimization algorithms require that the type of distribution is

[2] This aspect of portfolio allocation models is incorporated in the text of the PPA which requires "… generally accepted investment theories that take into account the historic returns of different asset classes over defined periods of time, …"

made explicit. Several such types have been used in the literature and in forecasting models.[3]

### *Diversification*

Perhaps the most robust and important implication of investment theory is the benefit of diversification. Because different investments do well at different times, a portfolio constructed of multiple assets will generate portfolios with lower risk for a given level of return than individual assets. However, not all risk can be diversified away. Elton et al. (2007) showed that 73 percent of the risk of an individual security in the U.S. market can be eliminated by holding a random portfolio of stocks. The risk that can be diversified away is referred to as idiosyncratic risk, while systematic or undiversifiable risk is based on components common among all assets and cannot be removed from the portfolio.

The effect of diversification can be even larger when additional asset classes are introduced, such as foreign stocks or real estate investment trusts. The less correlated these asset classes are with existing classes, the greater the potential reduction in risk. Even further improvements to an investor's opportunities exist when these classes are less correlated with labor income.[4]

Many 401(k) programs offer stock and bond mutual funds that are constructed to provide significant diversification. The benefits of further diversification through the investment in multiple diversified investment vehicles prevalent in 401(k) funds may be of limited additional benefit and lead to higher administrative costs. However, the prescription of diversification has several important implications. First, portfolios with heavy weighting in a single asset, or set of assets with correlated returns, will not be optimal. While an allocation of 100 percent of the portfolio into a single diversified fund may be optimal, an allocation of a substantial portion of the portfolio into a single asset can most likely be improved upon by introducing additional assets. Second, models may seek to weight the portfolio in asset classes where there is a low correlation with other invested assets and with labor income. As an example, models may seek to invest in higher-risk portfolios with lower relative return if these portfolios are foreign and therefore less correlated with labor income. As another example, it is less likely optimal for a homeowner with high exposure to property value fluctuations to invest in a real estate investment trust than for an investor without exposure to real estate. This last example highlights an important point: the benefits of portfolio diversification apply widely to all assets of an investor, not just to his 401 (k) portfolio.

### *Capital Asset Pricing Model (CAPM)*

The Capital Asset Pricing Model (CAPM), credited to Sharpe (1964) and Lintner (1965), is a widely-used application of the Markowitz model. The CAPM involves a few

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[3] Oftentimes, rates of returns are assumed to follow a multivariate normal distribution. However, some researchers have found that rates of return tend to have higher probabilities of very small and very large returns than embedded in normal distributions (e.g., Fama, 1965) and some portfolio allocation models accommodate alternative distributions.

[4] By contrast, assets whose rates of return are highly correlated with labor income, such as shares in the company in which the investor works, would generate additional idiosyncratic risk that can be diversified away.

additional assumptions on the structure of returns, investment opportunities (such as borrowing, short-selling, and the availability of a risk-free asset), and investor preferences. It relies on the construction of a set of available assets that, combined with some amount of the risk-free asset, creates a set of investments that provide the maximum level of return for any risk exposure, or the minimum level of risk for any return level. Under the assumptions of the CAPM, investors would all hold the same portfolio of risk-bearing investments, which would be combined in varying proportions with the risk free asset, depending on the individual investors desired level of risk and expected return. Since all investors will hold the same basket of risky assets, equilibrium conditions require that this is a portfolio of all securities, weighted by their value. In the real world, no such portfolio is available or priced, as this would include all assets, including non-financial assets. Models and investment strategies often use proxies such as the S&P 500 portfolio or other value-weighted market averages.

While the CAPM has a number of limitations, it is widely used as the basic framework for analyzing portfolio allocations. Much of the finance literature is devoted to extensions of the CAPM which enhance its ability to mimic actual investor behavior. These potential causes include transaction costs, limits to borrowing, non-standard investor time horizons, investor preferences that do not conform to standard mean-variance models, and various other structural issues.

***Intertemporal Capital Asset Pricing Model and Arbitrage Pricing Theory***

A commonly used extension of the CAPM is the Intertemporal Capital Asset Model (ICAPM), proposed by Merton (1973). While the CAPM is concerned with only wealth as a measure of future consumption, the ICAPM looks at additional "state variables." These variables represent future states of the world that will affect the consumption decision. State variables may include relative price changes and the distribution of future asset returns conditional on future events. As an example, an investor may be worse off when the economy goes into a recession, and as a result, he may prefer investments that pay off well in this state of the world (Cochrane, 2001). Such a result is beyond the scope of the original CAPM, which considers only the current and the next time period.

The Arbitrage Pricing Theory (APT), developed by Ross (1976), is based on the law of one price, which states that assets with identical payouts in the future must trade at the same price today. It uses this insight to price assets based on factor portfolios that explain the returns of the stocks. The APT makes no use of absolute pricing, as all pricing is relative to the factor portfolios. The APT's empirical implications are similar to those of an ICAPM model.

Following the availability of computational power and data, more recent exercises have been undertaken to find factor portfolios which explain APT returns or mimic state variables in an ICAPM. The most commonly used variables in the literature are a hypothetical portfolio formed by buying small capitalization stocks and financing this by short-selling large capitalization stocks and a hypothetical portfolio formed by buying high-book-to-market value stocks and financing this by short-selling low-book-to-market value stocks. Often, the size portfolio is referred to as SMB (Small Minus Big) and the book-to-market portfolio as HML (High Minus Low) or VMG (Value Minus Growth).

These factors, derived in Fama and French (1993), in addition to the market portfolio, explain much of the overall cross-sectional variance in stock returns.

### *Efficient Markets*

A dominant theme in the finance literature is that of efficient markets. If markets are efficient, security prices fully reflect all available information. Efficient markets are based on the theory that investors on average use the information available in the market so that market prices and investment opportunities can not be reliably exploited to make abnormal profits. Any mispricings due to incomplete use of information in the market are immediately corrected by arbitrageurs. Fama (1970) surveyed the literature citing explanations for, and evidence of, efficient capital markets.

The efficient market hypothesis is hotly debated in the literature. Proponents argue that arbitrageurs will capture any excess profit opportunities immediately, and that strategies to exploit inefficiencies will not generate excess returns. Instead, examples of exceptional historical performance are attributed to random chance, luck, or measurement error. Proponents of behavioral finance argue that arbitrage is not sufficient in the market to eliminate abnormal profit opportunities because it is costly or risky. Further, they argue that investors are not fully rational and can cause mispricings to appear in the market through systematic action. They conclude that the market is not fundamentally efficient enough to eliminate at least some abnormal profit opportunities.[5]

Some hedge funds and other investors attempt to develop strategies based on market inefficiencies. Since there is no consensus in the literature over market efficiency and the merits of investment strategies to exploit potential inefficiencies—especially after taxes and transactions costs—, we do not discuss them here (but see Barberis and Thaler, 2003, for examples and their associated costs and risks).

### *Life Cycle Theories*

The basic portfolio allocation model of Markowitz (1952) considers investment decisions over only two periods. Its allocation prescription applies without regard to age- or time-related factors. Life cycle theories of investment incorporate time, age, and wealth aspects. They focus on the level of investment over time and suggest how portfolio assets should shift between risky and risk-free assets over time. Life cycle theories may incorporate investor preferences for consumption and bequests, investor patience (trade-off between consumption today versus at a future date), and investor survival probabilities. They may further account for income, accumulated wealth, investment opportunities, and expected future income such as from Social Security.

Wallmeier and Zainhofer (2006) and Cocco, Gomes, and Maenhout (2005) discussed a basic life cycle model and derived the following general conclusions.

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[5] For more on the efficient markets debate, see Fama (1991) for an efficient markets perspective and Barberis and Thaler (2003) for a behavioral perspective. For surveys of the existing literature on market efficiency and behavioral finance see Malkiel (2003) and Shiller (2003). While differing in opinion on whether markets are broadly efficient, both authors conclude that attempts to exploit inefficiencies cannot be expected to yield consistently extraordinary returns.

- The optimal degree of riskiness of one's portfolio decreases in age until retirement. More specifically, the investor is generally fully invested in risky assets until his early thirties, and his optimal exposure to risky assets falls to approximately 50 percent of investments at retirement for the average investor. The drivers behind this finding are that older workers are closer to retirement than younger workers and that the probability of survival to retirement increases with age. Both factors imply that it is more important for older workers than for younger workers to have sufficient retirement wealth.
- At higher ages, investors with excess wealth may increase their exposure to risky assets because of declining probabilities of survival and very low risk of outliving their resources. This may cause investors to be willing to risk more as they are less sensitive to reduction in their future wealth, as the probability that it will be needed has decreased.

While these findings are robust to many standard specifications, Ameriks and Zeldes (2004) and other empirical studies have found that investors increase their exposure to risky assets through approximately age 50, and then reduce risks. This hump-shaped distribution is often seen in empirical studies, both in the United States and abroad. The explanation for this empirical regularity may lie in a positive correlation between stock market returns and future labor income (Bodie, Merton, and Samuelson, 1992). Young investors face large and uncertain future income flows and thus limit their exposure to similar risks in the stock market; with age, future labor income becomes smaller and the correlation with stock market returns less pronounced, so that the optimal portfolio moves into stocks; close to retirement, the standard argument above starts to dominate and the worker's portfolio should move to less risky assets.

Life cycle patterns in optimal portfolio allocations may also result from relatively novel features such as a strong desire to achieve or maintain a threshold level of consumption (keeping up with the Joneses, habit formation), allowances for borrowing, reducing the ability to withdraw from retirement savings, and the consideration of such other assets as housing and their contribution to the income stream.

## 3. Summary and Conclusion

The foundations for modern investment theories were laid by Markowitz (1952), who showed that portfolio diversification can reduce riskiness while preserving expected returns or increase expected returns without taking on more risk. In stripped-down form, the widely used CAPM even implies that every investor should hold a fully diversified portfolio consisting of every asset. The CAPM remains the basic framework that most scholars and practitioners use to explain investor choices and derive optimal portfolios. Many attempts have been made to improve on the CAPM, including the ICAPM, APT, and life cycle theories. The latter are particularly relevant to 401(k) participants, because the objectives, survival probabilities, and labor incomes of workers/investors change with age, so that their optimal portfolios also change with age.

A key assumption underlying most investment theories is that asset markets are efficient. However, there is no consensus over the extent to which this assumption holds. Some investment strategies aim to identify and exploit market inefficiencies, but as suggested by the lack of consensus, such strategies are controversial.

## 4. Disclaimer

This memo describes widely-used investment theories and models. The determination of whether widely-used investment theories are "generally accepted" is somewhat arbitrary. There is broad consensus among scholars and practitioners about the mathematics of optimal portfolio allocation. There is some diversity of opinion regarding which theories and models more closely reflect the reality of market behavior. Many of these extensions require significant additional effort to implement and may result in additional risks. The additional costs and risks associated with these more precise models may outweigh the financial benefit produced by their increased precision. Therefore, some extensions of the basic framework may be generally accepted as more precise yet not in widespread use, whereas the assumptions underlying some widely-used strategies to exploit market inefficiencies are not generally accepted as being the most precise reflections of market behavior. In addition as the knowledge of markets and operational methods advance some models which were not considered advantageous to implement may become "generally accepted" for practical use. In the absence of objective criteria, precisely what is and what is not generally accepted is therefore subject to interpretation and may evolve over time.

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## Appendix

The views, opinions, and/or findings contained in this report are those of the authors and should not be construed as an official Government position, policy or decision, unless so designated by other documentation.

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